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$N = 2$ Supersymmetric Ward Identities and Renormalized BRS and Anti-BRS Operators in Harmonic Superspace

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The quantization of $N = 2$ supersymmetric Yang-Mills theory coupled to a Fayet-Sohnius hypermultiplet is performed in the harmonic superspace, by requiring BRS and anti-BRS invariances. The corresponding Ward identities and the renormalized BRS and anti-BRS symmetries are derived.

INTRODUCTION

Supersymmetric field theories have been attracting much interest, since in such theories there are remarkable cancellations of ultraviolet divergences. These cancellations are expected to be even more dramatic in the case of extended supersymmetry. So far, superfield perturbation theory has been developed only for $N = 1$ supersymmetry (Salam and Strathdee, 1974, 1975). The cancellations of ultraviolet divergences are most easily seen in this context of superfield perturbation theory, where supersymmetry is manifest (Wess and Zumino, 1974; Iliopoulos and Zumino, 1974; Howe *et al.*, 1984). Indeed, quadratic divergences are absent, a fact which gives a key to resolving the hierarchy problem (Veltman, 1980). However, in the supersymmetric models the cancellations are due to the supersymmetric Ward identities (WI) and the breaking of these identities leads to additional ultraviolet divergences in the perturbative theory.

In order to handle the ultraviolet divergences in a systematic way for extended supersymmetry, it is apparently necessary to develop the superfield formulation. Galperin *et al.* (1984) put forward a new idea called harmonic superspace and developed $N = 2$ superspace formulations. Their method has the advantage in that their superfields are not constrained. The main feature of the harmonic superspace is that it contains a *Zweibein* $U_i^{\pm}$ $(i = 1, 2)$

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which parametrizes the coset space $SU(2)/U(1)$ and any superfield in general contains an infinite number of component fields (Galperin *et al*, 1984; Siegel, 1985).

Superfield perturbation methods for this theory in the harmonic superspace have been developed in two ways. The first is by using the Faddeev-Popov procedure where the quantum Lagrangian is obtained via covariant gauge (Galperin *et al.*, 1985*a*; Ohta and Yamaguchi, 1985). The second is by requiring the BRS and anti-BRS invariance (Lhallabi and Saidi, 1986, 1988; Lhallabi, 1988*a*), which is the clue to the proof of renormalizability.

In this paper we construct the $N=2$ supersymmetric quantum Lagrangian of the gauge and matter hypermultiplets using the BRS and anti-BRS prescription. The gauge fixing term is obtained by using the equation of motion of the auxiliary superfield b. For a particular choice of gauge parameters, one recovers the extended R-ξ gauge in $N=2$ supersymmetric theory. Furthermore, the $N=2$ supersymmetric WI are obtained by using simultaneously the BRS and anti-BRS symmetries. Following Zinn-Justin (1984), the renormalized $N=2$ supersymmetric Lagrangian in harmonic superspace can be obtained. However, some difficulties specific to the harmonic superspace, for instance, the infrared divergences in the kinetic term of V^{++}, make the fulfillment of this renormalization method not straightforward. Indeed, it is argued that these may be regulated in the context of dimensional regularization (Mandelstam, 1983; Brink *et al.*, 1983; Johansen, 1986) and do not affect the validity of the WI.

The paper is organized as follows. In Section 2 we give some preliminaries on $N=2$ harmonic superspace, while in Section 3 we derive the BRS and anti-BRS transformations for gauge and matter hypermultiplets. Then we construct the $N=2$ supersymmetric, BRS and anti-BRS invariant Lagrangian corresponding to gauge and matter hypermultiplets. However, we show that for the R-ξ gauge choice we obtain an inconvenient mixing of gauge and matter hypermultiplets as in $N=1$ and $N=0$ Yang-Mills theories interacting with the matter multiplet. In Section 4 we derive the $N=2$ supersymmetric WI by using simultaneously the BRS and anti-BRS symmetries. The action of WI operators on the effective action leads to local insertion operators. These latter are written as integrals with the full $N=2$ Grassmann measure. Furthermore, the renormalized BRS and anti-BRS operators leaving invariant the renormalized Lagrangian are obtained. Finally, Section 5 is devoted to our conclusions.

2. GENERALITIES ON THE $N=2$ HARMONIC SUPERSPACE

We begin this section by briefly recalling some concepts of harmonic analysis, following the conventions and notations of Galperin *et al.* (1984), Lhallabi and Said (1986, 1988), and Lhallabi (1988*a*).

In the harmonic superspace (HS) formalism, the harmonic variables (*Zweibein*) $U_i^{\pm}$ play an important role in the construction of N-extended supersymmetric theories. They are introduced as nonlinear realization of the $SU(2)$ group with $U^c(1)$ as a stability subgroup (Galperin *et al.*, 1984) and satisfy the following properties:

$$U^{+i}U_i^- = 1, \qquad U^{\pm i}U_i^{\pm} = 0 \tag{2.1}$$

They have an $SU(2)$ index i and a $U^c(1)$ index $\pm$. With their help, any $SU(2)$ isospinor F^i can be converted into a pair of independent $U^c(1)$ objects:

$$F^{\pm} = F^i U_i^{\pm} \tag{2.2}$$

The $N=2$ harmonic superspace in the real or central basis (CB) is parametrized by the coordinates $(X^{\mu}, \theta_{\alpha}^i, \bar{\theta}_{\dot{\alpha}}^i, U_i^{\pm})$, where θ_{α}^i and $\bar{\theta}_{\dot{\alpha}}^i$ are Weyl spinors and $SU(2)$ isospinors. From (2.2) one can pass to another basis called the analytic basis (AB) given by

$$z = (X_A^{\mu} = X^{\mu} - 2i\theta^{(i}\sigma^{\mu}\bar{\theta}^{j)}U_i^+U_j^-, \theta_{\alpha}^+, \bar{\theta}_{\dot{\alpha}}^+, \theta_{\alpha}^-, \bar{\theta}_{\dot{\alpha}}^-, U_i^{\pm}) \tag{2.3}$$

In this (AB) the $N=2$ supersymmetry is realized as follows:

$$\begin{aligned} \delta X_A^{\mu} &= -2i(\varepsilon^i\sigma^{\mu}\bar{\theta}^+ + \theta^+\sigma^{\mu}\bar{\varepsilon}^i)U_i^- \\ \delta\theta_{\alpha}^{\pm} &= \varepsilon_{\alpha}^i U_i^{\pm} \\ \delta\bar{\theta}_{\dot{\alpha}}^{\pm} &= \bar{\varepsilon}_{\dot{\alpha}}^i U_i^{\pm} \\ \delta U_i^{\pm} &= 0 \end{aligned} \tag{2.4}$$

We note that the variable sets $Z_A = (X_A^{\mu}, \theta_{\alpha}^+, \bar{\theta}_{\dot{\alpha}}^+, U_i^{\pm})$ and $(X_R^{\mu} = X^{\mu} + i(\theta^+\sigma^{\mu}\bar{\theta}^- - \theta^-\sigma^{\mu}\bar{\theta}^+), \bar{\theta}_{\dot{\alpha}}^+, \bar{\theta}_{\dot{\alpha}}^-, U_i^{\pm})$ form closed subsets under the $N=2$ supersymmetric transformations. We call these analytic and chiral subspaces (AS) and (CS), respectively. This allows us to define an analytic superfield (ASF) $\phi^q(X_A, \theta^+, \bar{\theta}^+, U^{\pm})$ satisfying the analyticity condition:

$$D^+\phi^q = 0 = \bar{D}^+\phi^q$$

where q is the $U^c(1)$ charge and

$$D^+ = \partial/\partial\theta^-, \qquad \bar{D}^+ = \partial/\partial\bar{\theta}^- \tag{2.5}$$

In this (AS), the covariant derivatives are defined by

$$D^{++} = \partial^{++} - 2i\theta^+\sigma^{\mu}\bar{\theta}^+\partial_{\mu}^A \tag{2.6}$$

where

$$\begin{aligned} \partial^{++} &= U^{+i}\frac{\partial}{\partial U^{-i}} \\ D^- &= -\frac{\partial}{\partial\theta^+} + 2i\sigma^{\mu}\bar{\theta}^-\partial_{\mu}^A \\ \bar{D}^- &= -\frac{\partial}{\partial\bar{\theta}^+} - 2i\theta^-\sigma^{\mu}\partial_{\mu}^A \end{aligned} \tag{2.7}$$

It is well known from the study of irreducible representations of N-extended supersymmetric algebra (Wess and Bagger, 1983; Gates *et al.*, 1983; Ferrara and Savoy, 1982) that there exist two global $N=2$ multiplets, denoted as $(0^4, 1/2^2)$ and $(0^2, 1/2^2, 1)$. The gauge multiplet $(0^2, 1/2^2, 1)$ is described by the real analytic superfield V^{++} having two $U^c(1)$ charges and zero mass dimension. The corresponding field strength W, which is a chiral superfield, independent of $U_i^{\mp}$ variables, leads to the classical action (Galperin *et al.*, 1985*a*; Ohta and Yamaguchi, 1985)

$$I_0=\int d^4X_R\, d^2\bar{\theta}^+\, d^2\bar{\theta}^+ d^2\bar{\theta}^- \operatorname{Tr} W^2 \tag{2.8}$$

which can be expanded as

$$I_0=\int d^4X_A\, dU\, \mathscr{L}_0$$

where

$$\mathscr{L}_0=\int d^2\theta^+\, d^2\bar{\theta}^+ \operatorname{Tr}\left[V^{++}\frac{1}{D^{++}}\frac{(D^+)^4D^{--}}{16}V^{++}\right]+\mathscr{L}_{\text{int}}(V^{++}) \tag{2.9}$$

with

$$D^{--}=U^{-i}\frac{\partial}{\partial U^{+i}}-2i\theta^-\sigma^\mu\bar{\theta}^-\partial_\mu^A$$

For the $N=2$ scalar multiplet $(0^4, 1/2^2)$ there corresponds two analytic superfields, the Fayet-Sohnius (FS) hypermultiplet (Fayet, 1976; Sohnius, 1978) and the Howe-Stelle-Townsend (HST) hypermultiplet (Howe *et al.*, 1983). The first one is described by a complex ASF ϕ^+ having one $U^c(1)$ charge and one mass dimension. The second one is obtained from a non-charged, real ASF Ω. However, the free action describing the (FS) multiplet ϕ^+ is given by

$$I_\phi=\int d^4X_A\, dU\, \mathscr{L}_\phi \tag{2.10a}$$

with

$$\mathscr{L}_\phi=\int d^2\theta^+\, d^2\bar{\theta}^+\, \overset{*}{\phi}{}^+ D^{++}\phi^+ \tag{2.10b}$$

Furthermore, the analytic superfields V^{++} and ϕ^+ transform under gauge transformation of any gauge group G as

$$V^{++\prime}=-ie^{i\Lambda}(D^{++}+igV^{++})e^{-i\Lambda} \tag{2.11a}$$

$$\phi^{+\prime}=e^{i\Lambda}\phi^+ \tag{2.11b}$$

where Λ is a real, analytic superfield.

It has been established (Galperin, 1984) that gauge interactions can be obtained from free global actions (2.10) by covariantizing the harmonic covariant derivatives D^{++}, namely

$$D^{++} \to d^{++} = D^{++} + igV^{++} \tag{2.12}$$

Therefore, the full action of the (FS) multiplet interacting with the gauge hypermultiplet is given by

$$I'_\phi = \int d^4X_A\, dU\, \mathscr{L}'_\phi, \qquad \mathscr{L}'_\phi = \int d^2\theta^+\, d^2\bar{\theta}^+\, \overset{*}{\tilde{\phi}}{}^+\, d^{++}\phi^+ \tag{2.13}$$

Now we construct the $N=2$ supersymmetric quantum Lagrangian describing gauge and matter hypermultiplets by requiring BRS and anti-BRS invariances.

3. $N=2$ SUPERSYMMETRIC QUANTUM LAGRANGIAN

In order to construct the quantum Lagrangian of $N=2$ supersymmetric Yang–Mills theory interacting with the (FS) matter hypermultiplet, we will postulate that the independent superfields are the gauge multiplet V^{++}, the analytic ghost and antighost superfields C and C' respectively, the auxiliary superfield b, and the (FS) hypermultiplet. All these superfields are in the adjoint representation of a compact group G. The quantization of this theory can be achieved by adding a gauge fixing term to the Lagrangian (2.9) (Galperin *et al.*, 1985*a*; Ohta and Yamaguchi, 1985) or by requiring BRS and anti-BRS symmetries (Lhallabi and Saidi, 1986, 1988; Lhallabi, 1988*a*). Such symmetries may be understood as being the necessary conditions for the cancellation between the unphysical and ghost modes. They are deduced by extending the (AS) to

$$y_A = (Z_A, \xi, \xi') \tag{3.1}$$

where ξ, ξ' are anticommuting scalar variables. A superfield in this extended analytic subspace (EAS) may be expanded as

$$\begin{aligned}\tilde{\phi}^9(y_A) &= \phi^9(Z_A) + \xi\delta_\xi\tilde{\phi}^9(y_A)|_{\xi=\xi'=0} + \xi'\delta_{\xi'}\tilde{\phi}^9(y_A)|\gamma_{\xi=\xi'=0} \\ &\quad + \xi\xi'\delta_\xi\delta_{\xi'}\tilde{\phi}^9(y_A)|_{\xi=\xi'=0}\end{aligned} \tag{3.2}$$

The BRS (δ_ξ) and anti-BRS ($\delta_{\xi'}$) transformations for ghost superfields C, C' and the auxiliary superfield b are obtained from the flatness in the unphysical directions ξ and ξ' by (Lhallabi and Saidi, 1986, 1988; Lhallabi, 1988*a*)

$$\begin{aligned} &\delta_\xi C = -C\times C, \qquad && \delta_{\xi'} C = -b + C\times C' \\ &\delta_\xi C' = b && \delta_\xi C' = -C'\times C' \\ &\delta_\xi b = 0 && \end{aligned} \tag{3.3}$$

where

$$A \times B = \tfrac{1}{2}[A, B]$$

For the gauge hypermultiplet V^{++} and (F.S) hypermultiplet ϕ^+, the BRS and anti-BRS transformations are obtained by making a special gauge transformation with a gauge parameter $\tilde{\Lambda}$ which is restricted to be an analytic superfield:

$$\tilde{\Lambda} = \xi C + \xi' C' + \xi\xi' b \tag{3.4}$$

Hence, the gauge transformation (2.12) can be extended as

$$\begin{aligned} \tilde{V}^{++} &= -ie^{i\tilde{\Lambda}}(D^{++} + igV^{++})e^{-i\tilde{\Lambda}} \\ \phi^+ &= e^{i\tilde{\Lambda}}\phi^+ \end{aligned} \tag{3.5}$$

However, for an infinitesimal transformation one has

$$\begin{aligned} \delta V^{++} &= ig[\tilde{\Lambda}, V^{++}] - D^{++}\tilde{\Lambda} \\ \delta\phi^+ &= i\tilde{\Lambda}\phi^+ \end{aligned} \tag{3.6}$$

Using equation (3.2), one has for the variations of V^{++} and ϕ^+

$$\begin{aligned} \delta V^{++} &= \xi\delta_\xi V^{++} + \xi'\delta_{\xi'} V^{++} + \xi\xi'\delta_\xi\delta_{\xi'} V^{++} \\ \delta\phi^+ &= \xi\delta_\xi\phi^+ + \xi'\delta_{\xi'}\phi^+ + \xi\xi'\delta_\xi\delta_{\xi'}\phi^+ \end{aligned} \tag{3.7}$$

Identifying (3.6) and (3.7), one obtains the BRS and anti-BRS transformations for V^{++} and ϕ^+, namely

$$\delta_\xi V^{++} = -D^{++}C - ig[V^{++}, C]; \qquad \delta_{\xi'} V^{++} = -D^{++}C' - ig[V^{++}, C'] \tag{3.8a}$$

$$\delta_\xi\phi^+ = iC\phi^+; \qquad \delta_{\xi'}\phi^+ = iC'\phi^+ \tag{3.8b}$$

As can be seen, the classical Lagrangian

$$\begin{aligned} \mathscr{L}_{\text{cl}} = &\int d^2\theta^+\, d^2\bar{\theta}^+\, \mathrm{Tr}\left[V^{++}\frac{1}{D^{++}}\frac{(D^+)^4 D^{--}}{16}V^{++}\right] \\ &+ \int d^2\theta^+\, d^2\bar{\theta}^+\, \overset{*}{\bar{\phi}}{}^+(D^{++} + igV^{++})\phi^+ + \mathscr{L}_{\text{int}}(V^{++}) \end{aligned} \tag{3.9}$$

is invariant with respect to the BRS and anti-BRS symmetry equations (3.8), (3.8b). However, in order to quantize the theory, we shall add to the classical

Lagrangian (3.9) variations with respect to ξ and ξ'. They read

$$\mathscr{L}_Q=\mathscr{L}_{\rm cl}+\frac{1}{2}\int d^2\theta^+\,d^2\bar{\theta}^+\operatorname{Tr}\Big\{\delta_\xi\delta_{\xi'}\Big[V^{++}V^{++}$$
$$+32\alpha C'(D^{++})^2\frac{1}{(D^+)^4(D^{--})^2}C\Big]\Big\}$$
$$+\int d^2\theta^+\,d^2\bar{\theta}^+\,\delta_\xi\delta_{\xi'}\{v^{3+}\phi^++\bar{v}^{3+}\bar{\overset{*}{\phi}}{}^++\lambda^{2+}\bar{\overset{*}{\phi}}{}^+\phi^++\cdots\}\tag{3.10}$$

where α is a gauge parameter and v^{3+} and λ^{2+} are dimensioned coupling constants. The dots indicate that higher order interaction terms are allowed. Furthermore, note in (3.9) the nonlocality of the V^{++} kinetic term in the harmonic variables. These harmonic nonlocalities are shown to disappear in supergraph calculations (Galperin *et al.*, 1985*b*). In contrast, the second term in equation (3.10) is nonlocal in X space. This is related to the dimension (+2) of the measure and the dimension (+1) of ghost superfields.

Let us now show how a gauge fixing term can be obtained in this way. To this end, one computes the $\delta_\xi\delta_{\xi'}$ term; using (3.3) and (3.8), restricting ourselves to the written terms, one obtains

$$\mathscr{L}_Q=\mathscr{L}_{\rm cl}+\int d^2\theta^+\,d^2\bar{\theta}^+\operatorname{Tr}\Big\{D^{++}C'\mathscr{D}^{++}C+g^2[V^{++},C][V^{++},C']$$
$$-\frac{\alpha}{4}[C',C']\frac{(D^{++})^2}{\Box}[C,C]+v^{3+}C'C\phi^++\bar{v}^{3+}\bar{\overset{*}{\phi}}{}^+CC'$$
$$-2\lambda^{2+}\bar{\overset{*}{\phi}}{}^+[C,C']\phi^++bD^{++}V^{++}-\frac{\alpha}{2}b\frac{(D^{++})^2}{\Box}b$$
$$+\frac{\alpha}{8}b\frac{(D^{++})^2}{\Box}[C,C']+ib(v^{3+}\phi^++\bar{v}^{3+}\bar{\overset{*}{\phi}}{}^++2\lambda^{2+}\bar{\overset{*}{\phi}}{}^+\phi^+)\Big\}\tag{3.11}$$

where $\mathscr{D}^{++}C=D^{++}C+ig[V^{++},C]$.

Eliminating the auxiliary superfield b by using its equation of motion, one gets the following form for the quantum Lagrangian:

$$\mathscr{L}_Q=\mathscr{L}_{\rm cl}+\int d^2\theta^+\,d^2\bar{\theta}^+\operatorname{Tr}\Big\{D^{++}C'\mathscr{D}^{++}C$$
$$+\frac{1}{\alpha}V^{++}\Big[\Box V^{++}-2\Box\frac{1}{D^{++}D^{--}}V^{++}$$
$$+2i\frac{\Box}{D^{++}}(v^{3+}\phi^++\bar{v}^{3+}\bar{\overset{*}{\phi}}{}^+)+4i\frac{\Box}{D^{++}}\lambda^{2+}\bar{\overset{*}{\phi}}{}^+\phi^+\Big]$$
$$+v^{3+}C'C\phi^++\bar{v}^{3+}\bar{\overset{*}{\phi}}{}^+CC'-2\lambda^{2+}\bar{\overset{*}{\phi}}{}^+[C,C']\phi^++g^2[V^{++},C][V^{++},C']$$
$$+\frac{\alpha}{4}[C',C']\frac{(D^{++})^2}{\Box}[C,C]\text{ higher order in }v^{3+},\lambda^{2+}\Big\}\tag{3.12}$$

where

$$\frac{1}{\alpha}\left[V^{++}\Box V^{++}-V^{++}\frac{2\Box}{D^{++}D^{--}}V^{++}\right.$$
$$\left.+2iV^{++}\frac{\Box}{D^{++}}(v^{3+}\phi^{+}+\bar{v}^{3+}\overset{*}{\bar{\phi}}{}^{+})+4iV^{++}\frac{\Box}{D^{++}}\lambda^{2+}\overset{*}{\bar{\phi}}{}^{+}\phi^{+}\right] \tag{3.13}$$

is the gauge-fixing term. When $v^{3+}=\lambda^{2+}=0$ one recovers the gauge condition $D^{++}V^{++}=0$ containing the Lorentz gauge of $N=0$ Yang-Mills theory (Galperin *et al.*, 1985*a*). Furthermore, for $v^{3+}\neq 0$ and $\lambda^{2+}=0$ one obtains the $N=2$ supersymmetric $R_{-\xi}$ gauge, namely

$$f^{4+}=D^{++}V^{++}+\frac{i}{\xi}\frac{1}{\Box}[a^{+}D^{++}\phi^{+}+\bar{a}^{+}D^{++}\overset{*}{\bar{\phi}}{}^{+}] \tag{3.14}$$

with

$$v^{3+}=\frac{1}{\xi}a^{+}\frac{D^{++}}{\Box}$$

where a^{+} is a constant and ξ is a new gauge parameter.

In fact, if we consider the following gauge-fixing Lagrangian term of the Faddeev-Popov theory (Galperin *et al.*, 1985*a*),

$$\mathscr{L}_{\rm gf}=-\frac{1}{\alpha}\int d^2\theta^{+}\,d^2\bar{\theta}^{+}\,\mathrm{Tr}\,\overset{*}{\bar{f}}{}^{4+}\frac{1}{(D^{++})^2}\frac{(D^{+})^4(D^{--})^2}{16}f^{4+} \tag{3.15}$$

and if we use (3.14), we obtain

$$\mathscr{L}_{\rm gf}=\frac{1}{\alpha}\int d^2\theta^{+}\,d^2\bar{\theta}^{+}\,\mathrm{Tr}\left\{V^{++}\Box V^{++}-V^{++}\frac{2\Box}{D^{++}D^{--}}V^{++}\right.$$
$$\left.+2iV^{++}[a^{+}\phi^{+}+\bar{a}^{+}\overset{*}{\bar{\phi}}{}^{+}]+\text{higher order in } a^{+}\right\} \tag{3.16}$$

which is analogous to the gauge fixing term (3.13). Furthermore, equation (3.16) shows that this gauge leads to an inconvenient mixing of V^{++} and ϕ^{+} as the $N=1$ supersymmetric (Ovrut and Wess, 1982) and ordinary Yang-Mills theories (Baulieu and Thierry Mieg, 1982) interacting with matter fields. We expect that such coupling can be overcome if the internal symmetry is spontaneously broken. Indeed, one may ask if there exists any $N=2$ supersymmetric scalar potential for which the gauge symmetry is broken. The answer to this question will be given elsewhere.

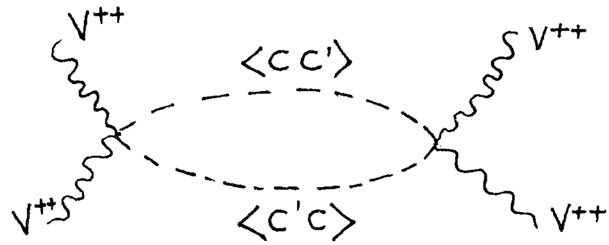

Fig. 1

However, as a matter of fact, the differences in the Feynman rules between the Faddeev–Popov theory (Galperin *et al.*, 1985*a*; Ohta and Yamaguchi, 1985; Ovrut and Wess, 1982) and the BRS and anti-BRS invariant theory are: (1) the ghost-gauge superfield coupling is changed, and (2) the fourth-order ghost–ghost coupling, which is analogous to that in the Curci and Ferrari (1976) form of the $N=0$ Yang–Mills Lagrangian, appears at the tree level. Such a term is nonlocal in X space and disappears for the $U(1)$-gauge group. Furthermore, knowing the $N=2$ supersymmetric Faddeev–Popov theory results, the only new diagrams are those displayed in Figures 1 and 2.

Now we come to the derivation of $N=2$ supersymmetric WI of BRS and anti-BRS symmetries by restricting ourselves, for simplicity, to the pure Yang–Mills case.

4. THE $N=2$ SUPERSYMMETRIC WARD IDENTITIES AND RENORMALIZATION

As we have seen in Section 3, the quantization of $N=2$ supersymmetric Yang–Mills theory has much in common with the case of $N=0$. However,

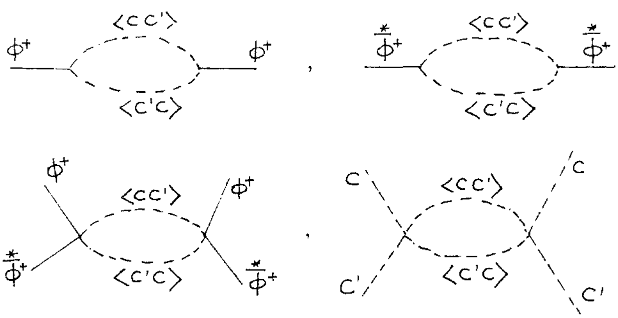

Fig. 2

the renormalization method of the underlying theory can be made in the same way as that given in Zinn-Justin (1974), Baulieu and Thierry Mies (1982), and Baulieu (1985) for ordinary Yang-Mills theory. Indeed, some difficulties specific to the harmonic superspace and to the BRS and anti-BRS quantization method, namely (1) the harmonic nonlocality ($\alpha \neq 1$) in the kinetic and gauge-fixing terms of V^{++} and (2) the nonlocal fourth order ghost-ghost coupling equation in (3.12), make the fulfillment of this program very delicate. Therefore, the BRS and anti-BRS quantization will suffer from infrared divergences. It is argued that these may be regulated in the context of dimensional regularization (Mandelstam, 1983; Brink *et al.*, 1983; Johansen, 1986) and do not affect the validity of the WI. In what follows, we derive the $N=2$ supersymmetric WI by using simultaneously the BRS and anti-BRS symmetries. We shall also confirm that all the quantum corrections to the effective action can be written as integrals with the full $N=2$ Grassmann measure.

Let us start from the pure Yang-Mills $N=2$ supersymmetric and gauge-fixed Lagrangian, namely

$$\begin{aligned}L_{\mathrm{Q}}=\int d^2\theta^+\,d^2\bar{\theta}^+\,dU\,\mathrm{Tr}\Big\{&\frac{1}{\alpha}\,V^{++}\Box V^{++}+\left(1-\frac{1}{\alpha}\right)V^{++}\frac{1}{D^{++}}\frac{(D^+)^4}{16}D^{--}V^{++}\\&+D^{++}C'\mathscr{D}^{++}C+g^2[V^{++},C][V^{++},C']-\frac{\alpha}{4}[C',C']\frac{(D^{++})^2}{\Box}[C,C]\\&+bD^{++}V^{++}-\frac{\alpha}{2}b\frac{(D^{++})^2}{\Box}b+\frac{\alpha}{8}b\frac{(D^{++})^2}{\Box}[C,C']\Big\}+L_{\mathrm{int}}(V^{++})\end{aligned}\tag{4.1}$$

Furthermore, consider a system of external analytic superfields $\rho_q'^{2+}$ and $\eta_q'^{4+}$ (q indicates the Faddeev-Popov charge) coupled to composite superfields as follows

$$\begin{aligned}L=L_{\mathrm{Q}}+\int d^2\theta^+\,d^2\bar{\theta}^+\,dU\,\{&\rho_{-1}'^{2+}\delta_\xi V^{++}+\eta_{-2}'^{4+}\delta_\xi C+\eta_0'^{4+}\delta_\xi C'\\&+\rho_{+1}'^{2+}\delta_{\xi'}V^{++}+\eta_0'^{4+}\delta_{\xi'}C+\eta_{+2}'^{4+}\delta_{\xi'}C'+\rho_0'^{2}+\delta_\xi\delta_{\xi'}V^{++}\\&+\eta_{-1}'^{4+}\delta_\xi\delta_{\xi'}C+\eta_{+1}'^{4+}\delta_\xi\delta_{\xi'}C'\}\end{aligned}\tag{4.2}$$

This Lagrangian can be written as a full $d^8\theta$ integral by using the identities

$$\begin{aligned}F^q&=\frac{(D^+)^4(D^{--})^2}{32\Box}F^q\qquad\text{for } V^{++},\ C,\ C',\ \text{and } b\\\rho_q'^{2+}&=\frac{(D^+)^4}{16}\rho_q^{2-}\\\eta_q'^{4+}&=\frac{(D^+)^4}{16}\eta_q\end{aligned}\tag{4.3}$$

for external analytic superfields. Hence (4.2) becomes

$$L = L_Q + \int d^8\theta\, dU\{\rho_{-1}^{2-}\delta_\xi V^{++} + \eta_{-2}\delta_\xi C + \eta_0\delta_\xi C'$$
$$+\rho_{+1}^{2-}\delta_{\xi'} V^{++} + \eta_0\delta_{\xi'} C + \eta_{+2}\delta_{\xi'} C' + \rho_0^{2-}\delta_\xi\delta_{\xi'} V^{++}$$
$$+\eta_{-1}\delta_\xi\delta_{\xi'} C + \eta_{+1}\delta_\xi\delta_{\xi'} C'\} \tag{4.4}$$

where

$$L_Q = \int d^8\theta\, dU\, \mathrm{Tr}\left\{\frac{1}{2\alpha} V^{++}(D^{--})^2 V^{++} + \left(1 - \frac{1}{\alpha}\right) V^{++}\frac{D^{--}}{D^{++}} V^{++}\right.$$
$$-\frac{1}{2}\frac{(D^{--})^2}{\Box} C' D^{++}\mathscr{D}^{++} C + \frac{g^2}{2}\left[V^{++}, \frac{(D^{--})^2}{\Box} C\right][V^{++}, C']$$
$$-\frac{\alpha}{8}\left[C', \frac{(D^{--})^2}{\Box} C'\right]\frac{(D^{++})^2}{\Box}[C, C] + \frac{1}{2}\frac{(D^{--})^2}{\Box} bD^{++}V^{++}$$
$$\left.-\frac{\alpha}{4}\frac{(D^{--})^2}{\Box} b\frac{(D^{++})^2}{\Box} b + \frac{\alpha}{16}\frac{(D^{--})^2}{\Box} b\frac{(D^{++})^2}{\Box}[C, C']\right\} + L_{\mathrm{int}}(V^{++}) \tag{4.5}$$

In order to avoid infrared problems in harmonic variables, we choose the supersymmetric Feynman gauge ($\alpha = 1$) usually preferred in the literature due to the better infrared behavior of the propagators. Furthermore, if we introduce external nonanalytic sources coupled to analytic superfields, the Lagrangian (4.5) becomes

$$L' = L + \int d^8\theta\, dU\, \{J_v^{2-} V^{++} + J_c C + J_{c'} C' + J_b b\} \tag{4.6}$$

Note that the BRS and anti-BRS invariances are maintained by prescribing that all external superfields do not transform. The generating functional of Green functions is then given by

$$\exp[iW(J, \rho, \eta)]$$
$$= \int DV^{++}\, DC\, DC'\, Db \exp\left\{i \int d^4x\, L\right.$$
$$\left. + i\int d^{12}z\, dU\, [J_V^{2-} V^{++} + J_C C + J_{C'} C' + J_b b]\right\} \tag{4.7}$$

From (4.2) and (4.7) we remark that we may express all composite superfields by differentiation with respect to the corresponding *nonanalytic* external superfields. Moreover, the generating functional of one-particle irreducible is defined by

$$\Gamma[V^{++}, C, C', b, \rho, \eta'] = W[J, \rho, \eta] - \int d^{12} z\, dU[J_v^{2-} V^{++} + J_C C + J_{C'} C' + J_b b] \tag{4.8}$$

Now let us perform the infinitesimal change of all superfields $\phi \to \phi + \varepsilon\delta_\xi\phi$ and $\phi \to \phi + \varepsilon'\delta_{\xi'}\phi$; we obtain

$$0 = \int d^{12}z\, dU\{J_v^{2-}\delta_\xi V^{++} + J_C\delta_\xi C + J_{C'}\delta_{\xi'} C' + \rho_{+1}^{2-}\delta_\xi\delta_{\xi'} V^{++} + \eta_0\delta_\xi\delta_{\xi'} C + \eta_{+2}\delta_\xi\delta_{\xi'} C'\} \tag{4.9}$$

and

$$0 = \int d^{12}z\, dU\{J_V^{2-}\delta_{\xi'} V^{++} + J_C\delta_{\xi'} C + J_{C'}\delta_{\xi'} C' + J_b\delta_{\xi'} b + \delta_{-1}^{2-}\delta_\xi\delta_{\xi'} V^{++} + \eta_{-2}\delta_\xi\delta_{\xi'} C\} \tag{4.10}$$

or equivalently

$$W_\Gamma^{+1}\Gamma = \int d^{12}z\, dU\left\{\frac{1}{2}\left[\frac{\delta\Gamma}{\delta V^{++}}\frac{\delta}{\delta\rho_{-1}^{2-}} + \frac{\delta\Gamma}{\delta\rho_{-1}^{2-}}\frac{\delta}{\delta V^{++}}\right] + \frac{1}{2}\left[\frac{\delta\Gamma}{\delta C}\frac{\delta}{\delta\eta_{-2}} + \frac{\delta\Gamma}{\delta\eta_{-2}}\frac{\delta}{\delta C}\right] + b\frac{\delta}{\delta C'} + \rho_{+1}^{2-}\frac{\delta}{\delta\rho_0^{2-}} + \eta_0\frac{\delta}{\delta\eta_{-1}} + \eta_{+2}\frac{\delta}{\delta\eta_{+1}}\right\}\Gamma = 0 \tag{4.11a}$$

and

$$W_\Gamma^{-1}\Gamma = \int d^{12}z\, dU\left\{\frac{1}{2}\left[\frac{\delta\Gamma}{\delta V^{++}}\frac{\delta}{\delta\rho_{+1}^{2-}} + \frac{\delta\Gamma}{\delta\rho_{+1}^{2-}}\frac{\delta}{\delta V^{++}}\right] + \frac{1}{2}\left[\frac{\delta\Gamma}{\delta C}\frac{\delta}{\delta\eta_0} + \frac{\delta\Gamma}{\delta\eta_0}\frac{\delta}{\delta C}\right] + \frac{1}{2}\left[\frac{\delta\Gamma}{\delta C'}\frac{\delta}{\delta\eta_{+2}} + \frac{\delta\Gamma}{\delta\eta_{+2}}\frac{\delta}{\delta C'}\right] + \frac{1}{2}\left[\frac{\delta\Gamma}{\delta b}\frac{\delta}{\delta\eta_{+1}} + \frac{\delta\Gamma}{\delta\eta_{+1}}\frac{\delta}{\delta b}\right] - \rho_{-1}^{2-}\frac{\delta}{\delta\rho_0^{2-}} + \eta_{-2}\frac{\delta}{\delta\eta_{-1}}\right\}\Gamma = 0 \tag{4.11b}$$

These expressions (4.11) are the $N=2$ supersymmetric WI corresponding to BRS and anti-BRS invariances. We observe that the WI operators $W_\Gamma^{\mp 1}$ act nonlinearly on Γ. Such nonlinearity originates from the coupling of all external superfields to nonlinear BRS and anti-BRS variations of analytic superfields. Furthermore, the WI operators satisfy algebraically the following nilpotency properties as in the $N=0$ (Baulieu and Thierry Mieg, 1982) and $N=1$ (Lhallabi, 1986) Yang-Mills theories:

$$W_\Gamma^{+1} W_\Gamma^{+1}\Gamma = (W_\Gamma^{+1} W_\Gamma^{-1} + W_\Gamma^{-1} W_\Gamma^{+1})\Gamma = W_\Gamma^{-1} W_\Gamma^{-1}\Gamma = 0 \tag{4.12}$$

which play a fundamental role in the construction of a renormalized generating functional. However, in order to obtain a renormalized effective action, the WI must be satisfied to all orders of perturbation theory. If they cannot be satisfied, even though we add suitable finite local counterterms, then the symmetry is anomalous. Moreover, an important argument in working with renormalized quantities is the quantum action principle (Schwinger, 1951), which does not depend upon the details of a particular *regularization scheme.* Therefore, the action of WI operators on the effective action leads to local insertion operators. These terms are written as full $d^8\theta$ integral. Furthermore, the renormalized effective action, which we will call Γ_r, can be constructed by absorbing these insertion operators in the action.

In what follows, we will show that there exist renormalized BRS and anti-BRS invariances which keep the renormalized theory invariant. However, at the tree approximation of Γ_r, one has

$$\begin{aligned}\Gamma_r^{(0)} = \int & d^4x\, L_r(V^{++}, C, C', b) \\ &+ \int d^{12}z\, dU\, \{\rho_{-1}^{2-} q_{1V^{++}}^{2+} + \rho_{+1}^{2-} R_{1V^{++}}^{2+} + \rho_0^{2-} P_{1V^{++}}^{2+} \\ &+ \eta_{-2} q_{2C} + \eta_0 R_{2C} + \eta_{-1} P_{3C} + \eta_{+2} q_{3C'} + \eta_0' R_{3C'} \\ &+ \eta_{+1} P_{3C'} + \eta_{+1} R_b + \mathcal{P}(\rho, \eta, V^{++}, C, C', b)\}\end{aligned} \tag{4.13}$$

where q_i, R_i, and $\mathcal{P}$ are unknown polynomials in the superfields and external superfields. By construction, $\Gamma_r^{(0)}$ must satisfy equations (4.11) and (4.12), namely

$$W_{\Gamma_r}^{+1}\Gamma_r^{(0)} = 0 = W_{\Gamma_r}^{-1}\Gamma_r^{(0)} \tag{4.14a}$$

$$\begin{aligned} W_{\Gamma_r}^{+1} W_{\Gamma_r}^{+1}\Gamma_r^{(0)} &= (W_{\Gamma_r}^{+1} W_{\Gamma_r}^{-1} + W_{\Gamma_r}^{-1} W_{\Gamma_r}^{+1})\Gamma_r^{(0)} \\ &= W_{\Gamma_r}^{-1} W_{\Gamma_r}^{-1}\Gamma_r^{(0)} = 0 \end{aligned} \tag{4.14b}$$

These equations are in fact sufficient to determine the explicit form of L_r and the unknown polynomials. Let us now introduce the following operators:

$$\begin{aligned} \delta_\xi^r &= \int d^{12}z\, dU \left\{ q_{1V^{++}}^{2+} \frac{\delta}{\delta V^{++}} + q_{2C}\frac{\delta}{\delta C} + b\frac{\delta}{\delta C'} \right\} \\ \delta_{\xi'}^r &= \int d^{12}z\, dU \left\{ R_{1V^{++}}^{2+} \frac{\delta}{\delta V^{++}} + R_{2C}\frac{\delta}{\delta C} + R_{3C'}\frac{\delta}{\delta C'} + R_b\frac{\delta}{\delta b} \right\} \\ (\delta_\xi^r \delta_{\xi'}^r) &= \int d^{12}z\, dU \left\{ P_{1V^{++}}^{2+} \frac{\delta}{\delta V^{++}} + P_{2C}\frac{\delta}{\delta C} + P_{3C'}\frac{\delta}{\delta C'} \right\} \end{aligned} \tag{4.15}$$

By definition,

$$\begin{array}{lll} \delta_\xi^r V^{++} = q^2_{1V^{++}}, & \delta_\xi^r C = q_{2C}, & \delta_\xi^r C' = b \\ \delta_{\xi'}^r V^{++} = R^{2+}_{1V^{++}}, & \delta_{\xi'}^r C = R_{2C}, & \delta_{\xi'}^r C' = R_{3C'} \\ (\delta_\xi^r \delta_{\xi'}^r) V^{++} = P^2_{1V^{++}}, & (\delta_\xi^r \delta_{\xi'}^r) C = P_{2C}, & (\delta_\xi^r \delta_{\xi'}^r) C' = P_{3C'} \\ \xi_\xi^r b = 0, & \delta_{\xi'}^r b = R_b, & (\delta_\xi^r \delta_{\xi'}^r) b = 0 \end{array} \tag{4.16}$$

Hence, equation (4.14a) are rewritten as

$$\delta_\xi^r \Gamma_r^{(0)} + \int d^{12}z\, dU \left\{ \frac{\delta\Gamma_r^{(0)}}{\delta V^{++}} \frac{\delta\mathcal{P}}{\delta\rho^{2-}_{-1}} + \frac{\delta\Gamma_r^{(0)}}{\delta C} \frac{\delta\mathcal{P}}{\delta\eta_{-2}} \right.$$
$$\left. \rho^{2-}_{+1} \frac{\delta\Gamma_r^{(0)}}{\delta\rho_0^{2-}} + \eta_0 \frac{\delta\Gamma_r^{(0)}}{\delta\eta_{-1}} + \eta_{+2} \frac{\delta\Gamma_r^{(0)}}{\delta\eta_{+1}} \right\} = 0 \tag{4.17a}$$

and

$$\delta_{\xi'}^r \Gamma_r^{(0)} + \int d^{12}z\, dU \left\{ \frac{\delta\Gamma_r^{(0)}}{\delta V^{++}} \frac{\delta\mathcal{P}}{\delta\rho^{2-}_{+1}} + \frac{\delta\Gamma_r^{(0)}}{\delta C} \frac{\delta\mathcal{P}}{\delta\eta_0} \right.$$
$$\left. + \frac{\delta\Gamma_r^{(0)}}{\delta C'} \frac{\delta\mathcal{P}}{\delta\eta_{+2}} + \frac{\delta\Gamma_r^{(0)}}{\delta b} \frac{\delta\mathcal{P}}{\delta\eta_{+1}} - \rho^{2-}_{-1} \frac{\delta\Gamma_r^{(0)}}{\delta\rho_0^{2-}} + \eta_{-2} \frac{\delta\Gamma_r^{(0)}}{\delta\eta_{-1}} \right\} = 0 \tag{4.17b}$$

Combining (4.13) and (4.17), one finds that the renormalized Lagrangian is δ_ξ^r and $\delta_{\xi'}^r$ invariant,

$$\delta_\xi^r L_r = 0 = \delta_{\xi'}^r L_r \tag{4.18}$$

and

$$(\delta_{\xi'}^r)^2 = (\delta_\xi^r \delta_{\xi'}^r + \delta_{\xi'}^r \delta_\xi^r) = (\delta_\xi^r)^2 = 0 \tag{4.19}$$

which must be verified for all values of external superfields. These equations in conjunction with

$$\delta_\xi^r C' = b \qquad \text{and} \qquad \delta_\xi^r b = 0 \tag{4.20}$$

are sufficient to determine the explicit action of δ_ξ^r and $\delta_{\xi'}^r$ on all superfields. However, since η_{-2} has dimension 2 and ghost number (-2), $\delta_\xi^r C$ must be bilinear in the ghost superfield C. Thus, there exists a renormalized bracket $[\cdot,\cdot]_r$ such that

$$\delta_\xi^r C = -\tfrac{1}{2} Z[C, C]_r \tag{4.21}$$

Furthermore, the condition (4.19) implies that $[\cdot,\cdot]_r$ satisfies a Jacobi identity. From the study of the cohomology of Lie algebra (Bandelloni *et al.*, 1978), $[\cdot,\cdot]_r$ is proportional to $[\cdot,\cdot]$. Let us call the proportionality constant $Z_1^{1/2} Z_g$:

$$[\cdot,\cdot]_r = Z_1^{1/2} Z_g [\cdot,\cdot] \tag{4.22}$$

For the case of V^{++}, by dimensions one has

$$\delta_\xi^r V^{++} = Z'(D^{++}C + i[V^{++}, C]_r) \tag{4.23}$$

The condition $(\delta_\xi^r)^2 V^{++} = 0$ leads to $Z = Z'$. In exactly the same way, we obtain

$$\begin{aligned} \delta_{\xi'}^r V^{++} &= Z''(D^{++}C' + i[V^{++}, C']_r) \\ \delta_{\xi'}^r C' &= -\tfrac{1}{2}Z''[C', C']_r \end{aligned} \tag{4.24}$$

where Z'' is *a priori* different from Z. For dimensional reasons, $\delta_{\xi'}^r C$ is given by

$$\delta_{\xi'}^r C = -\mu b + \nu[C, C']_r \tag{4.25}$$

but the condition $(\delta_\xi^r \delta_{\xi'}^r + \delta_{\xi'}^r \delta_\xi^r) V^{++} = 0$ implies that $\nu = -Z''$ and $Z'' = \mu Z$. Therefore, one gets

$$\delta_{\xi'}^r C = -\mu(b + Z[C, C']_r) \tag{4.26}$$

Consequently, by using the condition $(\delta_{\eta'}^r)^2 C = 0$, we obtain from (4.24)

$$\delta_{\xi'}^r b = -Z\delta_{\xi'}^r [C, C']_r$$

or equivalently

$$\delta_{\xi'}^r b = \mu Z[C', b]_r \tag{4.27}$$

Hence, the condition $(\delta_{\xi'}^r)^2 b = 0$ is immediately satisfied. Finally, δ_ξ^r and $\delta_{\xi'}^r$ satisfy the following equations:

$$\begin{aligned} \delta_\xi^r V^{++} &= Z(D^{++}C + i[V^{++}, C]_r), & \delta_{\xi'}^r V^{++} &= \mu Z(D^{++}C' + i[V^{++}, C']_r) \\ \delta_\xi^r C &= -\tfrac{1}{2}Z[C, C]_r, & \delta_{\xi'}^r C &= -\mu(b + Z[C, C']_r) \\ \delta_\xi^r C' &= b, & \delta_{\xi'}^r C' &= -\tfrac{1}{2}\mu Z[C', C']_r \\ \delta_\xi^r b &= 0, & \delta_{\xi'}^r b &= \mu Z[C', b]_r \end{aligned}$$

We note that equations (3.3), (3.8a), and (4.26) are identical up to an overall rescaling. Therefore, L_r can be determined, in the same way as (3.10), by requiring the δ_ξ^r and $\delta_{\xi'}^r$ invariances. This leads to the multiplicative renormalizability of $N = 2$ supersymmetric Yang-Mills theory.

5. CONCLUSION

In this paper we first derived the BRS and anti-BRS transformations for all superfields of $N = 2$ supersymmetric theory. Also we have seen how the matter hypermultiplet is included in the quantization of $N = 2$ supersymmetric Yang-Mills theory in harmonic superspace by requiring BRS and anti-BRS invariances. The $R_{-\xi}$ or t'Hooft gauge is recovered for a particular choice of gauge parameters. Second, we have derived the BRS and anti-BRS WI of $N=2$ supersymmetric, Yang-Mills theory by using the functional formalism. The renormalized effective action is then given by adding local counterterms in all superfields for the Feynman gauge $\alpha=1$. Furthermore, the renormalized BRS and anti-BRS operators, which will imply a multiplicative renormalizability of $N=2$ supersymmetric theory, have been obtained.

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